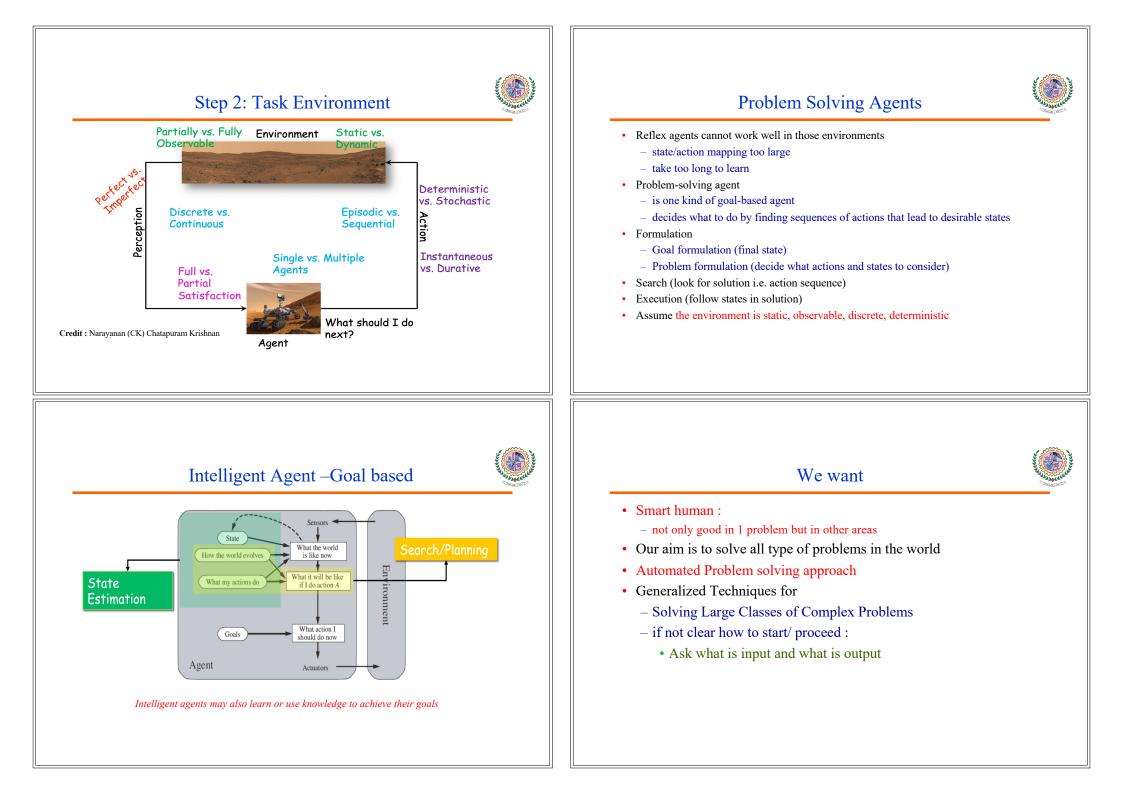
### **Artificial Intelligence** Module 2: Automated Problem Solving Module 2: Automated Problem Solving • PART 2.1: Intelligent Agent & Environment PART 2.3: Problem Representation in AI • PART 2.2: Complex Problems and AI • PART 2.3: Problem Representation in AI - Problem Solving Agents • Intelligent Agent - Problem Solving Methods • Defining the problem as a State Space Search Dr. Chandra Prakash 5 4 8 6 1 7 3 2 6 1 7 3 2 • Problem Decomposition Assistant Professor - Problem Formulation by AI Search Methods Department of Computer Science and Engineering • Modeling formulation of a problem to AI (Slides adapted from StuartJ. Russell, B Ravindran, Mausam, Prof. Pallab Dasgupta, Prof. Partha Pratim Chakrabarti, Saikishor Jangiti Type of Agents Design an Agent for The Mars Rover Sojourner • Step 1: Aim of the agent ???? • The 25-lb, 6-wheeled robotic explorer needs to Sensors State - travel on sandy, rocky terrain Simple reflex agents What the world How the world evolves - perceive its surroundings is like now Environment Model-based reflex agents - drive autonomously for short distances What it will be like What my actions do if I do action A - communicate and transmit data Goal-based agents - be remotely controlled How happy I will be in such a state Utility Utility-based agents - be transported easily - withstand extreme temperatures on Mars What action I should do now - carry necessary equipments

Agent

Actuators

- respond to events as they occur
- decide what to do next





### Problem solving agent : Pac-Man

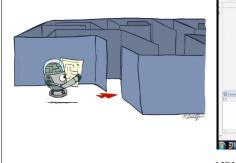


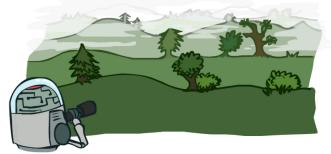


Figure out how to get to next dot. Clears board; non-optimal. Fast.

### Search Problems



### How to represent the environment/ world/ percepts for a problem ??



## Methods of Problem Representation in AI



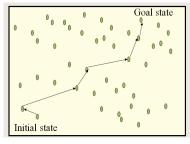
- Before a solution can be found, the prime condition is that the problem must be very precisely defined.
- The most common methods of problem representation in AI are:

### 1. State Space representation

- Includes the initial state S and all other states reachable from S by a sequence of actions
- 2. Problem Reduction
  - Whether the problem can be decomposed into smaller problems?
  - Using the technique of problem decomposition, we can solve very large problems easily.
    - Example for decomposable problems
      - $(x^2+3x+\sin 2x.\cos 2x) dx$

# Defining the problem: State Space representation

- State Space •
  - provide all possible state, operations and the goals.
  - all information about the environment
  - All information necessary to make a decision for the task at hand.
  - If the entire state-space representation for a problem is given, it is possible to trace the path from the initial state to the goal state and identify the sequence of operations necessary for doing it.
  - Limitation :
    - not possible to visualize all states for a given problem.

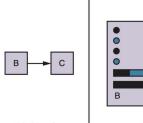


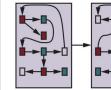
# Agent Classification in Terms of State Representations

Atomic St	tates are indivisible.	No internal structure Search on atomic
		states;
Propositional the the		Search+inference in logical (prop logic) and probabilistic (bayes nets) representations
Relational	tates describe the objects in the vorld and their interrelations	Search+Inference in predicate logic (or relational prob. Models)
<b>First-order</b> fu	unctions over objects	Search+Inference in first order logic (or first order probabilistic models)

### Spectrum of State Representations







(a) Atomic

(b) Factored

0

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(c) Structured

### Illustration with Vacuum World

**4** 

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-**A** 

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**4** 

**عم** 



### Atomic:

S1, S2.... S8, Each state is seen as an indivisible snapshot All Actions are SXS matrices.

If you add a second roomba the state space doubles If you want to consider noisiness of the rooms, the representation

Quadruples.

### **Relational:**

World made of objects: Roomba; L-room, R-room Relations: In (<robot>, <room>); dirty(<room>) If you add a second roomba, or more rooms, only the objects increase.

If you want to consider noisiness, you just need to add one other relation

### **Propositional/Factored:**

- States made up of 3 state variables
- Dirt-in-left-room T/F
- Dirt-in-right-room T/F
- Roomba-in-room L/R

Each state is an assignment of Values to state variables

23 Different states

Actions can just mention the variables they affect

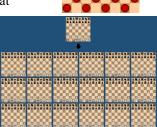
Note that the representation is compact (logarithmic in the size of the state space) If you add a second roomba, the

representation increases by just one more state variable.

If you want to consider "noisiness" of rooms, we need two variables, one for each room

	Representing States	La construction of the second s
•	<ul> <li>State space:</li> <li>A state space can be organized as a graph:</li> <li>nodes: states in the space</li> <li>arcs: actions/operations</li> </ul>	
•	<ul> <li>The size of a problem is usually described in terms the number of states (or the size of the state space) that are possible.</li> <li>Tic-Tac-Toe has about 3^9 states.</li> <li>Checkers has about 10^40 states.</li> </ul>	

- Chess has about 10<sup>120</sup> states in a typical game.
  - Shannon number



### Atomic Agent



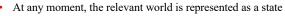
### • Input

- Set of states
- Operators [ and cost]
- Start state
- Goal state [Test]
  - This is a hard part that is rarely tackled in AI, usually assuming that the system designer or user will specify the goal to be achieved.

### • Output

- Path : start => a state satisfying goal test
- May require shortest path

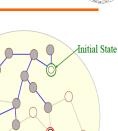
# Representing States



- Initial (start) state: S
- Possible action (or an operation)
  - changes the current state to another state (if it is applied):
- State transition / Transition Model
  - An action can be taken (applicable) only if the its precondition is met by the current state
  - For a given state, there might be more than one applicable actions
- Goal state/ Goal Test :
  - a state satisfies the goal description or passes the goal test Goal state
- Dead-end state:
  - a non-goal state to which no action is applicable

## Formalizing Search in a State Space

- A state space is a graph, (V, E) where
  - $-\ V$  is set of nodes and E is set of arcs
- Node: corresponds to a state
- Arc: corresponds to an applicable action/operation.
- node generation: making explicit a node by applying an action to another node which has been made explicit
- **node expansion:** generate **all** children of an explicit node by applying **all** applicable operations to that node
- One or more nodes are designated as start nodes
- A **goal test** predicate is applied to a node to determine if its associated state is a goal state
- A **solution** is a sequence of operations that is associated with a path in a state space from a start node to a goal node
- The cost of a solution is the sum of the arc costs on the solution path



Goal

## Problem Solving in AI



- A process or procedure used to find out the solution to a specific problem.
- To build a system to solve a particular problem we need to do 4 things.
  - 1. Define the problem precisely
    - This definition must include precise specifications of what the initial situations will be as well as what the final situations constitute acceptable solution to the problem.
  - 2. Analyze the problem
    - A few important features of the problem can help in the selection of various possible techniques for solving the problem.
  - 3. Isolate and represent the task knowledge that is necessary to solve the problem
  - 4. Choose the best problem-solving techniques and apply it to the particular problem.



OInitial state

State Space

0

0

0

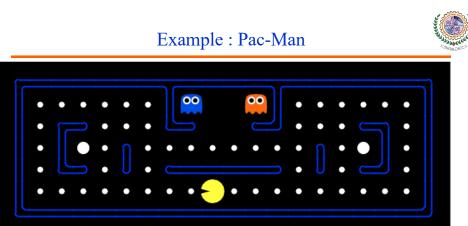
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0

### Problem Solving Stages



- 1. Assumptions
- 2. Solution steps
- 3. State Space Representations
  - Initial state
  - Final state
  - ✤ Operators that can be applied
  - Abbreviations
  - ✤ State space representation of the given problem
  - ✤ Operators and Conditions.



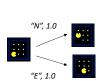
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## SCORE: 0

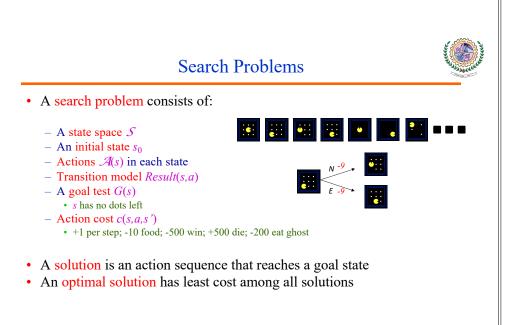
## Pac-Man Example : Search Problems



- A search problem consists of:
  - A state space
  - A successor function(with actions, costs)



- A start state and a goal test
- A solution is a sequence of actions (a plan) which transforms the start state to a goal state



## Pacman *agent program* in Python



### class GoWestAgent(Agent):

- def getAction(self, percept):
  - if Directions.WEST in percept.getLegalPacmanActions(): return Directions.WEST

### else:

return Directions.STOP

# Pac-Man Example: What's in a State Space?



### The world state includes every last detail of the environment

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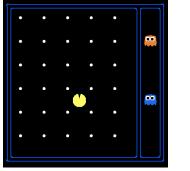
### A search state keeps only the details needed for planning (abstraction)

- Problem: Pathing
  - States: (x,y) location
  - Actions: NSEW
  - Successor: update location only
  - Goal test: is (x,y)=END
- Problem: Eat-All-Dots
  - States: {(x,y), dot booleans}Actions: NSEW
  - Successor: update location
  - and possibly a dot boolean
  - Goal test: dots all false



### • World state: 12x10 grid

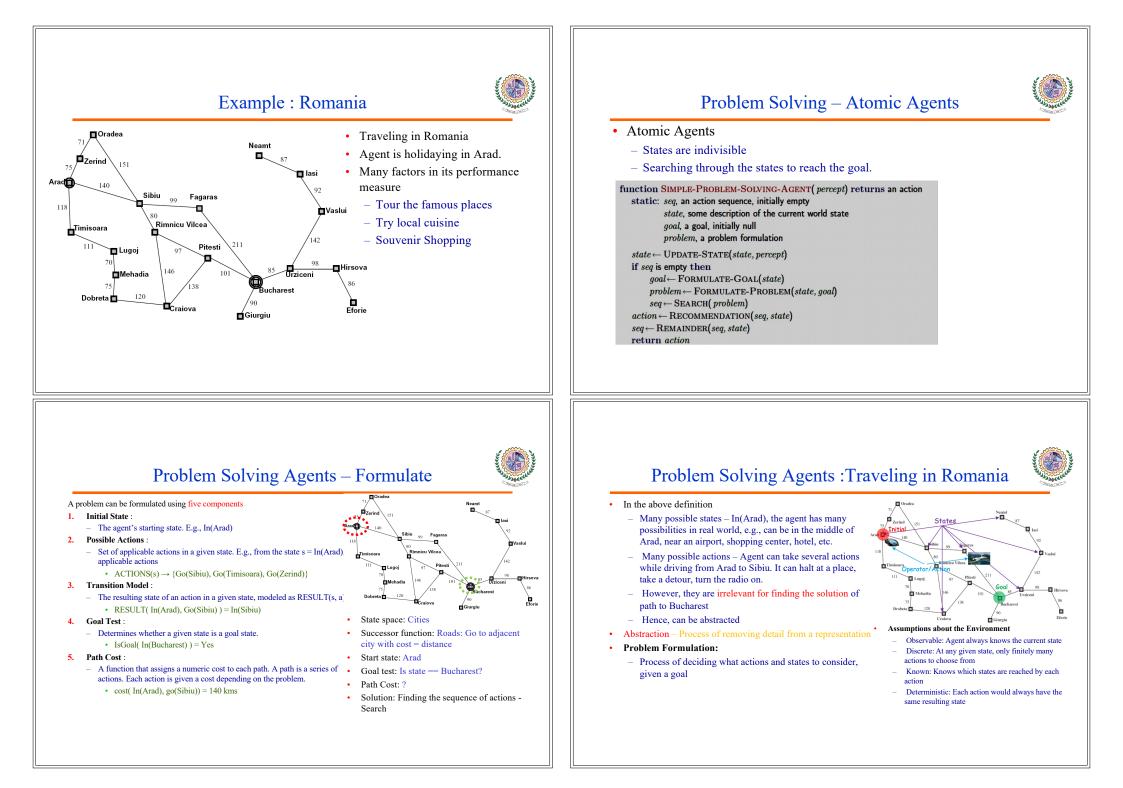
- Agent positions: 120
- Food count: 30
- Ghost positions: 12
- Agent facing: NSEW
- How many
  - World states?  $120x(2^{30})x(12^2)x4$
  - States for pathing?120
  - States for eat-all-dots?
     120x(2<sup>30</sup>)







- Problem: eat all dots while keeping the ghosts perma-scared
- What does the state space have to specify?
  - (agent position, dot booleans, power pellet booleans, remaining scared time)
- V



## Search Problems Are Models

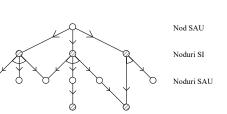




### AND/OR graph representation



- Problem decomposition into sub-problems
  - AND/OR graph
  - Solved node
  - Unsolvable node
  - Problem solution



## Problem Representation in AI



- Before a solution can be found, the prime condition is that the problem must be very precisely defined. The most common methods of problem representation in AI are:
  - 1. State Space representation
    - Includes the initial state S and all other states that are reachable from S by a sequence of actions
  - 2. Problem Reduction
    - Whether the problem can be decomposed into smaller problems?
    - Using the technique of problem decomposition, we can often solve very large problems easily.
      - Example for decomposable problems
      - $-\int (x^2 + 3x + \sin 2x \cdot \cos 2x) dx$

### Problem Formulation - Examples



- Toy Problems :
  - Intended to illustrate or exercise various problem solving methods. Useful in research for comparing the performance of algorithms
    - Vacuum World, 8-Puzzle, 8-Queens Problem,
    - · Cryptarithmetic, Missionaries and Cannibals
- Real-World Problems :
  - Problems for which solutions can be impacting people's daily lives.
    - Route finding, Touring problems
    - Traveling salesman problem,
    - VLSI layout, Robot navigation,
    - Assembly sequencing,
    - Protein Design, Internet Searching

## Modeling Formulate – Toy Problem Example



- **Possible States** There are two locations and two possibilities of dirt. Total of 8 possible states:
- $2x2^{2}=8$
- 1. State · Robot and dirt locations

  - Initial State - Any state with position of agent and dirt mentioned
- B

- Three possible actions, Left, Right, Suck 3. Transition Model
  - · Left would move the agent to left location, except when the agent is in leftmost location, there would be no effect.
  - · Similarly for Right action. Suck would remove the dirt, if any.
- 4 Goal Test

2. Actions

- · Checks whether all locations are clean or no Dirt
- 5. Path cost
  - Each step costs 1, so the path cost is total number of steps

2 3

6

8 7 5

Goal State

4

## Problem Formulation Example: 8-puzzle



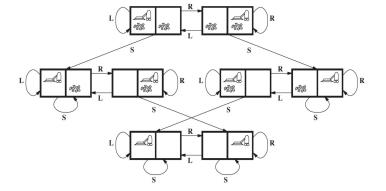


Initial State

- a) Initial state b) Final/ Goal State c) Operator

  - *i. Move(left)*
  - *ii. Move(right)*
  - *iii. Move(Up)*
  - iv. Move(Down)
  - Belongs to sliding-block puzzles
  - NP Complete problem.

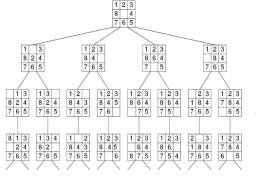
# Formalizing problem in a State Space: Toy Example



## Problem Formulation Example: 8-puzzle



- Possible states = 9!/2
- 1. Initial State :
  - Any permutation of 1-8 numbers with a blank
- 2. Actions :
  - Movement of blank space either by Left, Right, Up or Down
- 3. Transition Model :
  - The resulting state after moving the blank space will replace the digit
- Goal Test : 4.
  - Check whether the state matches the goal configuration
- 5. Path cost :
  - Each step costs 1, so the path cost is total number of steps



### Problem Formulation : Water Jug Problem

### **Define the Problem Accurately.**

- You are given two jugs, a 4 gallon one and a 3 gallon one.
- Neither has any measuring markers on it. There is a pump that can be used to fill the jugs with water.
- Discuss how exactly 2 gallon of water can be filled into 4 gallon of jug by employing the state space representations.



Goal: 2 Gallons Water in the 4 Gallon Juc

## Example : Step1-Assumptions



- In order to solve the given problem we can make the following assumptions without affecting the problem.
- 1. We can fill the jugs with the help of a pump.
- We can pour water from any jug to the ground. 2.
- We can transfer water from one jug to the other. 3.
- 4. No external measuring devices are available.

## Example : Step2- Solution Steps

State Space Representation

- Initial and final states 1.
- 2. Operators
- 3. Abbrivations
- 4. State space representation of the given problem State No. Operator

State (G4,G3)

- Operators and Conditions 5.
- 6. Production rules for Water Jug Problem

## State Space Representation

### 1. Initial & Final States

- The state space for this problem can be described as the set of ordered pairs of integers (x,y)
- such that x = 0, 1, 2, 3 or 4 and y = 0, 1, 2 or 3;
- x represents the number of gallons of water in the 4-gallon jug and
- y represents the quantity of water in 3-gallon jug
- The start state is (0,0)
- The goal state is (2,n) for any n.
- · Attempting to end up in a goal state. 2. Operators

Let us define the following 4 operators.



### Goal: 2 Gallons Water in the 4 Gallon Jud

- FILL (jug): where jug = 3 gallon or 4 gallon, fill the jug fully with water.
- EMPTY (jug): where jug = 3 gallon or 4 gallon, empty the jug by pouring the water to ground.
- POUR (jug1, jug2): pour the water from jug1 to jug 2 until it is just filled
- TRANSFER (jug1, jug2): pour the water from jug1 to jug 2 completely.

# State Space Representation

### 3. Abbreviations

G3 🗲 3 Gallon Jug

G4 🗲 4 Gallon Jug

### 4. State space representation of the given problem

State No.	Operator	State (G4,G3)
0 (Initial)		(0, 0)
1	FILL (G3)	(0, 3)
2	TRANSFER (G3, G4)	(3, 0)
3	FILL (G3)	(3, 3)
4	POUR (G3, G4)	(4, 2)
5	EMPTY (G4)	(0, 2)
6 (Final)	TRANSFER (G3,G4)	(2, 0)

# State Space Representation



Descritpion Fill the 4 gallon ju

Fill the 3 gallon jug Pour some water out of the 4 gallon ju

Pour some water out of the 3-gallon jug Empty the 4 gallon jug

Empty the 3 gallon jug on the ground Pour water from the 3 -gallon jug into the 4 -gallon jug until the 4-gallon jug is full

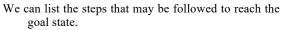
Pour all the water from the 3-gallon jug into the 4-gallon jug Pour all the water from the 4-gallon jug into the 3-gallon jug

Pour the 2 gallons from 3-gallon jug into the 4-gallon jug

Pour water from the 4-ga the 3-gallon jug is full

Operators	Conditions	SI No	Current state	Next Sta
Operators	Conditions	1	Current state (x,y) if x < 4	(4,y)
		2	(x,y) if y <3	(x,3)
FILL (G3)	: G3 was not full	3	$(x,y)$ if $x \ge 0$	(x-d, y)
	· C4	4	$(x,y)$ if $y \ge 0$	(x, y-d)
FILL (G4)	: G4 was not full	5	(x,y) if x>0	(0, y)
EMPTY (G4)	: G4 was not empty	6	(x,y) if y >0	(x,0)
	1 5	7	(x,y) if x+y >= 4 and y >0	(4, y-(4-
	~ -	8	(x, y) if x+y >= 3 and x>0	(x-(3-y),
RANSFER (G3,G4)	: G3 was not empty			
	& G4 was not fu	ıll	(x, y) if x+y <=4 and y>0	(x+y, 0)
		10	(x, y) if x+y <= 3 and x>0	(0, x+y)
OUR (G3, G4)	: G3 was not empty	11	(0,2)	(2,0)
	& G4 was not full			
	a G4 was not full	12	(2,y)	(0,y)

## Water Jug Problem : Solution

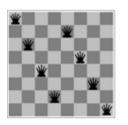


- 1. Fill the 3 gallon jug with water
- 2. Pour the water from 3 gallon jug to 4 gallon jug.
- 3. Again fill the 3 gallon jug with water
- 4. Pour the water carefully from 3 gallon jug to 4 gallon jug such that it is just filled.
- 5. Empty the 4 gallon jug
- 6. Transfer the water from 3 gallon jug to 4 gallon jug.
- 7. Stop.

	i	
Gallons in the 4-gallon jug	Gallons in the 3-gallon jug	Rule applied
0	0	2
0	3	9
3	0	2
3	3	7
4	2	5 or 12
0	2	9 0r 11
2	0	

### Problem Formulation: The 8-Queens

- Incremental formulation vs. complete-state formulation
- States-I-1: 0-8 queens on board
- Successor function-I-1:
  - add a queen to any square
  - # of possible states = (64\*63\*...\*57= )
- States-I-2:
  - 0-8 non-attacking queens on board
- Successor function-I-2:
  - add a queen to a non-attacking square in the left-most empty column
  - # of possible states = 2057 --- ???
- Goal test: 8 queens on board, none attacked
- Path cost: of no interest (since only the final state count)



## The Eight Queens Problem



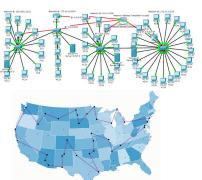
52

- One strategy: guess at a solution
  - There are 4,426,165,368 ways to arrange 8 queens on a chessboard of 64 squares
- An observation that eliminates many arrangements from consideration
  - No queen can reside in a row or a column that contains another queen
    - Now: only 40,320 (8!) arrangements of queens to be checked for attacks along diagonals

## Formulate – Real World Example

- Route finding problem, e.g., Google Maps
- Routing Video Streams in Computer Networks
- Traveling Salesman problem
- Robot Navigation





# N queens problem formulation

Formulation 2

### Formulation 1

- States: Any arrangement of 0 to States: Any arrangement of 8 queens on the board
  - Initial state: 0 queens on the board
- Successor function: Add a queen in any square
- Goal test: 8 queens on the board, none are attacked



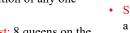
- 8 queens on the board - Initial state: All queens are at
- Successor function: Change
- queen



- board, none are attacked



- the position of any one
- Goal test: 8 queens on the



States: Any arrangement of k queens in the first k rows such that none are attacked

Formulation 3

- Initial state: 0 queens on the board
- Successor function: Add a queen to the (k+1)th row so that none are attacked.
- Goal test : 8 queens on the board, none are attacked 53

## Formulate – Real World Example

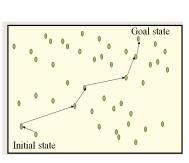


### Airline Travel problem

- the task of a travel agent to book cheapest and fastest flight route from City A to City B.
- E.g., Need to Travel from Delhi to Los Angeles within Rs. 70,000
- Possible States Each state is a location and current time.
  - 1. Initial State Specified by User's query
  - 2. Actions Flight from current location at a particular time with enough time for within-airport transfer if needed
  - Transition Model The destination location and arrival time 3.
  - 4. Goal test Are we at final destination at the specified time?
  - 5. Path Cost Flight cost + Duration + Waiting time + Immigration, etc.

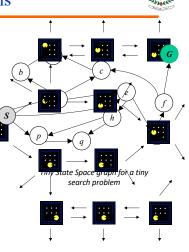
## Searching for Solutions

- Solution for above Problems:
  - A sequence of possible actions starting at the initial state and reaching the destination specified
- Search Algorithms:
  - Given problem formulation as input, these algorithms would output the sequence of actions
  - Search Trees
    - Where states are nodes and actions are edges. The initial state will be the root node and possible actions are branches



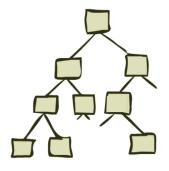
## State Space Graphs

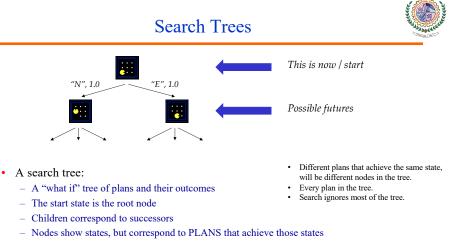
- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)
- In a state space graph, each state occurs only once!
- We can rarely build this full graph in memory (it's too big), but it's a useful idea
  - Find solution without writing most of the graph down.
  - really care about the start and what you can do from the start -> search tree



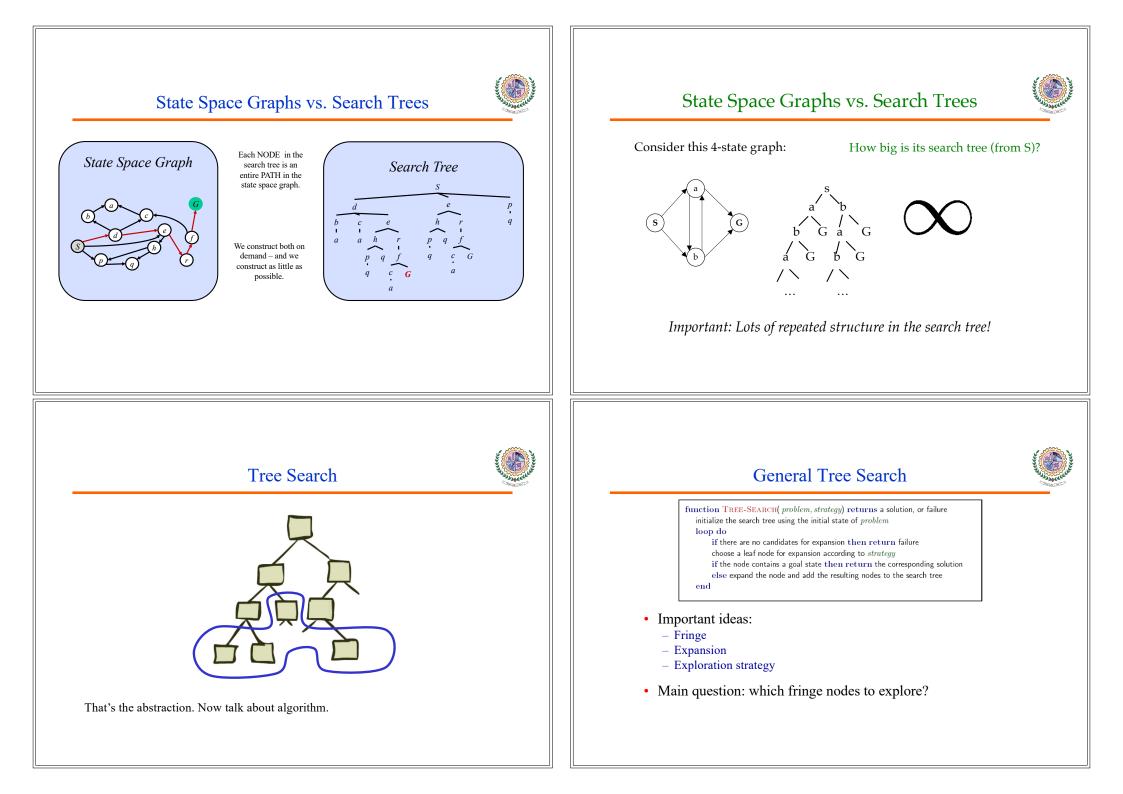
# State Space Graphs and Search Trees

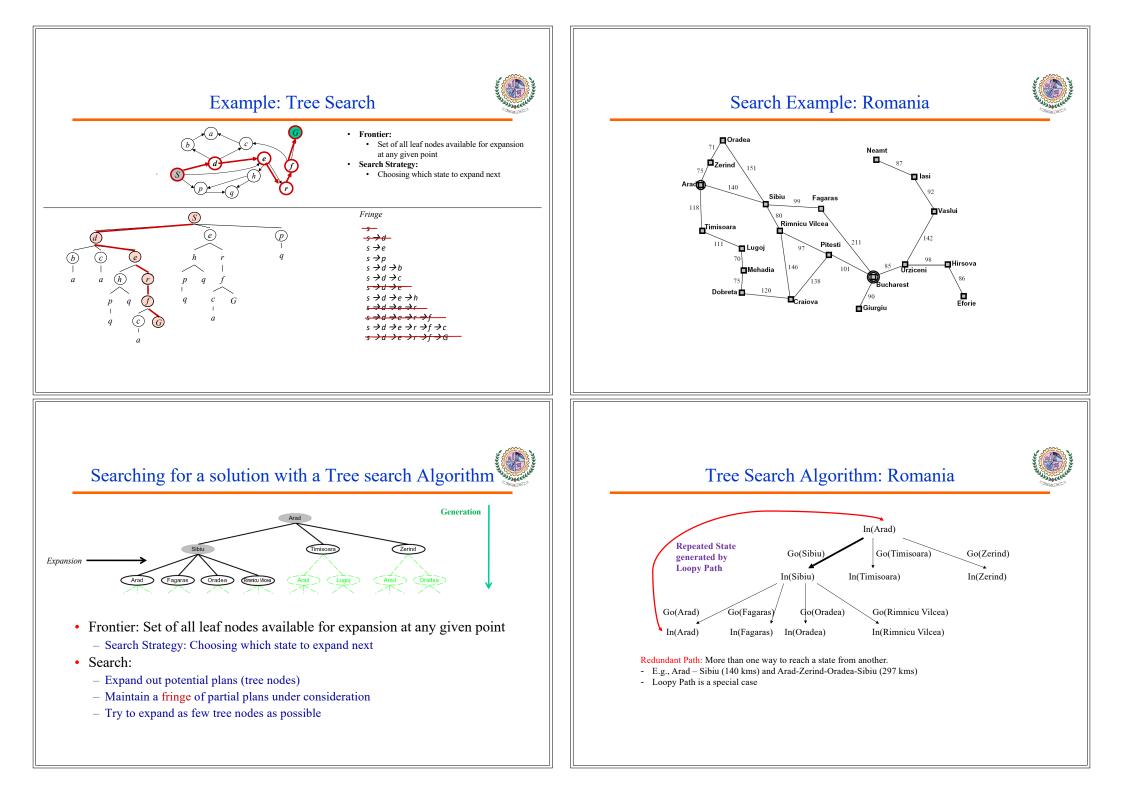






- For most problems, we can never actually build the whole tree





## Revision



- Description of the problem
  - Define a state space that contains all the possible configurations of the relevant objects.
  - Specify one or more states within that space that describe possible situations from which the problem solving process may start (initial state)
  - Specify one or more states that would be acceptable as solutions to the problem. (goal states)
  - Specify a set of rules that describe the actions (operations) available.
    - What are unstated assumptions?
    - How general should the rules be?
    - How much knowledge for solutions should be in the rules?

## Can Solution Steps be ignored or undone?



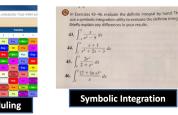
- Suppose we are trying to prove a math theorem.
  - We can prove a lemma. If we find the lemma is not of any help, we can still continue.
- 8-puzzle problem
- Chess: A move cannot be taken back.
- Important classes of problems:
  - Ignorable (e.g. theorem proving) in which solution steps can be ignored
  - Recoverable (e.g. 8-puzzle) in which solution steps can be undone.
  - Irrecoverable (e.g. chess) in which solution steps cannot be undone

## **COMPLEX PROBLEMS & SOLUTIONS**



## **COMPLEX PROBLEMS & SOLUTIONS**





## Measuring Search Algorithm Performance

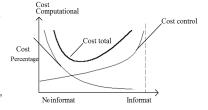


- Completeness :
  - Is the algorithm guaranteed to find a solution when there is one?
- Optimality :
- Does the algorithm find the optimal solution?
- Time Complexity :
  - How long does it take to find the solution?
- Space Complexity :
  - How much memory is needed to perform the search?
- Time and space complexity
  - Search in AI is represented by initial state, actions and transitions which usually result in infinite Nodes and Edges in a graph.Hence, the complexity is rather measured by
  - Branching Factor (b): Maximum number of successors of any node
  - Depth (d) of the shallowest goal, i.e., number of steps from initial node
  - Maximum length (m) of any path in state space ( may be  $\infty$  )

## Measuring Search Algorithm Performance



- Effectiveness of an algorithm can be measured by:
  - Search Cost:
    - Time spent by the algorithm in finding a solution (can also include memory usage)
  - Solution Cost:
    - Path cost of the solution
  - Total Cost:
    - Search Cost + Solution Cost (if not in same units, convert them)



## Search Algorithms



- Uninformed Search Algorithms
  - No additional information about states beyond what is provided in problem formulation
  - Generate successors and distinguish a goal state from a non-goal state
- Informed Search Algorithms
  - Strategies that know if one non-goal state is more promising than another non-goal state
  - Also called Heuristic Search strategies

Summary				A CONTRACTOR OF
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- Four steps for designing a program to solve a problem:
  - 1. Define the problem precisely
  - 2. Analyse the problem
  - 3. Identify and represent the knowledge required by the task
  - 4. Choose one or more techniques for problem solving and apply those techniques to the problem.

### Next : Explore • Problem Formulation by AI Search • Module 3: Search Strategies • # of states Methods for the following famous - PART 3.1: Search • State : problems : - PART 3.2: Uninformed Search • Initial State: - Tic-Tac-Toe - PART 3.3: Informed/Heuristic Search - 8-puzzel problem - PART 3.4: Beyond Classical Search • Action : - 8 Queens Problem – PART 3.5: Problem reduction • Goal test : - PART 3.6: Adversarial Search - Missionaries and Cannibals • path cost : - Tower of Hanoi - Travelling Salesman Problem (TSP) 2 - Rubik's Cube 5 6 з 4 5 8 3 Search Tree References · Artificial intelligence : A Modern Approach, Prentice Hall by Stuart Russell, Peter Norvig, Artificial Intelligence by Elaine Rich & Kevin Knight, Third Ed, Tata McGraw Hill • 2 З Artificial Intelligence and Expert System by Patterson Slides adapted from CS188 Instructor: Anca Dragan, University of California, Berkeley • 8 4 Slides adapted from CS60045 ARTIFICIAL INTELLIGENCE . 6 5