Artificial Intelligence

6.3 Fuzzy Logic Control

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Overview

- **Introduction**
- **Fuzzy Sets**
- **Linguistic Variables**
- **Membership Functions**
- **Fuzzy Logic Operators**
- **Fuzzy Control**
- **CASE Study**
- **Summary**

References

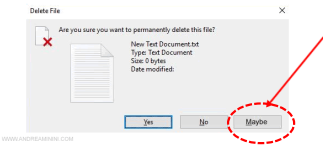
- L. Zadah, “Fuzzy sets as a basis of possibility” Fuzzy Sets Systems, Vol. 1, pp3-28, 1978.
- T. J. Ross, “Fuzzy Logic with Engineering Applications”, McGraw-Hill, 1995.
- K. M. Passino, S. Yurkovich, "Fuzzy Control" Addison Wesley, 1998.
- Intelligent Systems and Soft Computing
- Some slides are taken from “Introduction to Fuzzy Logic Control” by Andrew L. Nelson , University of South Florida

Introduction

- The sky is clear or cloudy?

- In the Boolean logic
  - 0 (clear sky) and
  - 1 (cloudy sky).
- More Appropriate- sky is also a little cloudy.

Introduction



- In computer science we use the Boolean logic which admits only two values: zero and one.
- All programming languages are based on the binary logic of George Boole.
  - A condition can be true or false, with no middle ground.
- No programmer would ever develop a user interface with the yes, no, and maybe buttons.

Introduction

- Boolean logic has a limited scope
- Reality is much more complex.
- In the Boolean logic the previous information ("a little cloudy") can not be transformed into data.
- In natural language, many **human expressions are subjective and relative**.
  - A Boolean machine could not understand them without falling into logical contradictions.
  - Experts rely on common sense when they solve problems.

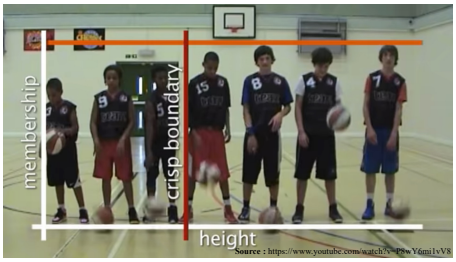
Example : Who is tall?

Source : https://www.youtube.com/watch?v=P8wY6mi1vV8

Example (cont…)

Source : https://www.youtube.com/watch?v=P8wY6mi1vV8

Example (cont…)



Source : https://www.youtube.com/watch?v=P8wY6mi1vV8

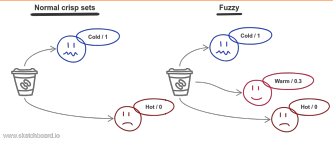
Boolean logic uses sharp distinctions. It forces us to draw lines between members of a class and non- members

Example

- How to define tall ?
- In fuzzy logic we can transform subjective information into an objective fact.

Set Vs Fuzzy Theory

- Set Theory
  - Binary
    - Hot
    - Cold



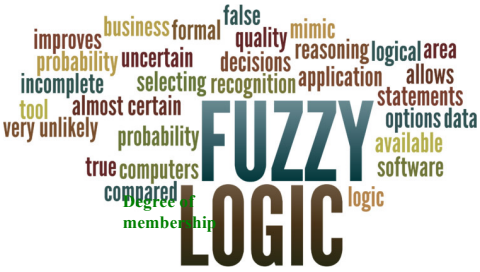
- Experiment error
  - e.g. 5 cm can be read as - 5.1 cm or 4.9 cm , etc..
- Experiment cant use crisp result from set theory
- Fuzzy logic is a **way to make use of natural language in logic**
  - **Degree of Truth**
  - **Degree of membership**

Fuzziness in Nature

Depth of Sea

- On the beach, you don't see the sea water ending in a perfectly straight line.
- Instead, **the depth of the sea gradually decreases.**
  - From shoulder length to
  - knee length to
  - wet toes then
  - moist sand and
  - finally fully dry sand.

Fuzzy Logic



Fuzzy Logic Introduction

- Fuzzy set theory by Lotfy Zadeh
- Fuzzy logic:
  - A way to represent variation or imprecision in logic
  - A way to make use of natural language in logic
  - Approximate reasoning
- Vagueness
- Fuzzy
- Probability
- Boundaries are vague/overlap in fuzzy
  - Low Temp | Medium Temp | High Temp
- Vagueness may not be fuzzy but fuzzy may be vagueness.

Fuzzy Logic

- Fuzzy logic reflects how people think.
- It attempts to model our sense of words, our decision making and our common sense.
  - more human, intelligent systems.
- How can we represent expert knowledge that uses vague and ambiguous terms in a computer
- Fuzzy logic is based on the idea that all things admit of degrees.
  - Temperature, height, speed, distance, beauty – all come on a sliding scale.
    - The motor is running really hot.
    - Tom is a very tall guy.

Crisp (Traditional) Variables

- $f_A(x)$: $X \to \{0, 1\}$, where $f_A(x) = \begin{cases} 1, \text{if } x \in A \\ 0, \text{if } x \notin A \end{cases}$
- Crisp variables represent precise quantities:
  - x = 3.1415296
  - A ∈ {0,1}
- A proposition is either True or False
  - **A ∧ B ⇒ C**
- King(Richard) ∧ Greedy(Richard) ⇒ Evil(Richard)
- Richard is either greedy or he isn't:
  - Greedy(Richard) ∈ {0,1}

Fuzzy Sets

- What if Richard is only somewhat greedy?
- Fuzzy logic is a set of mathematical principles for knowledge representation based on degrees of membership.
- Fuzzy Sets can represent the degree to which a quality is possessed.
- Fuzzy Sets (Simple Fuzzy Variables) have values in the range of [0,1]
- In the fuzzy theory, fuzzy set A of universe X is defined by function $\mu_A(x)$ called the ***membership function*** of set A

  $\mu_A(x)$: $X \to [0, 1]$, where $\mu_A(x) = 1$ if x is totally in A;
  $\mu_A(x) = 0$ if x is not in A;
  $0 < \mu_A(x) < 1$ if x is partly in A.
- Greedy(Richard) = 0.7 ,
- Question: How evil is Richard?

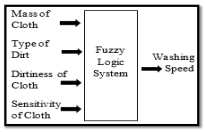
Fuzzy Sets : Example Who is tall?

Degree of membership

Real time Examples

Digital Camera

Washing Machine



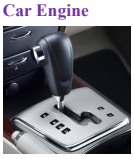
Input = dirty clothes, water, soap powder, electric
Output= dirty soapy water, clean clothes

Real time Examples

Networking Device

Airplane autopilot

Car Engine



Fuzzy Linguistic Variables

- Fuzzy Linguistic Variables are used to represent qualities spanning a particular spectrum
- Temp: {**Freezing**, **Cool**, **Warm**, **Hot**}
- Membership Function
  - Question: What is the temperature?
  - Answer: It is warm.
  - Question: How warm is it?

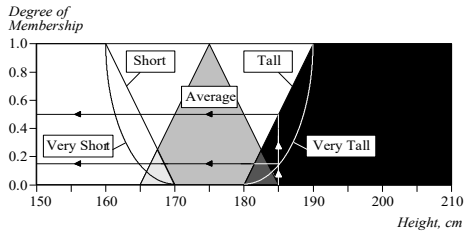
Linguistic variables and hedges

- At the root of fuzzy set theory lies the idea of linguistic variables.
- A linguistic variable is a fuzzy variable.
- For example, the statement "John is tall" implies that the linguistic variable John takes the linguistic value tall.
- The range of possible values of a linguistic variable represents the universe of discourse of that variable.
  - For example, the universe of discourse of the linguistic variable *speed* might have the range between 0 and 220 km/h and may include such fuzzy subsets as *very slow*, *slow*, *medium*, *fast*, and *very fast*.
- A linguistic variable carries with it the concept of fuzzy set qualifiers, called hedges.
  - Hedges are terms that modify the shape of fuzzy sets.
  - They include adverbs such as very, somewhat, quite, more or less and slightly.

Hedges

- Imagine that we have a function for tall such that tall(a) returns a's membership in the category
  - For instance, tall(5'2") = .1, tall(5'11") = .6 and tall(6'7") = .9
- A hedge is a fuzzy term that is used to convert the membership functions output
  - What does "very tall" mean? "somewhat tall"? "incredibly tall"?
- Common hedges:
  - Very → $f(x)^2$
  - Not very → $1 - f(x)^2$
  - Somewhat → $f(x)^{1/2}$
  - About (or around) → f(x) +/- delta
  - Nearly → f(x) – delta
- So for example
  - if our membership function for Old says that 52 is a old / .6
  - then 52 would be very old / .36, not very old .64, somewhat old / .77
- We would need to define a reasonable delta for things like "around"

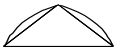
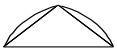
Fuzzy sets with the hedge very



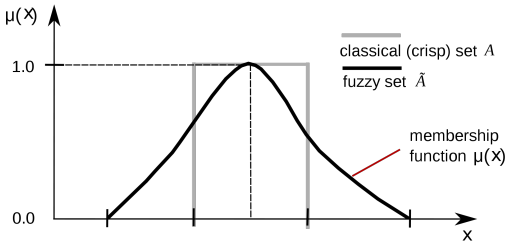
Representation of hedges in fuzzy logic

Hedge	Mathematical Expression	Graphical Representation
A Little more	$[\mu_A(x)]^{1.3}$	
Slightly more	$[\mu_A(x)]^{1.7}$	
Very	$[\mu_A(x)]^{2}$	
Extremely	$[\mu_A(x)]^{3}$	

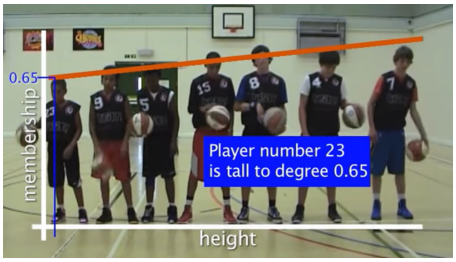
Representation of hedges in fuzzy logic

Hedge	Mathematical Expression	Graphical Representation
Very very	$[\mu_A(x)]^4$	
More or less	$\sqrt{\mu_A(x)}$	
Somewhat	$\sqrt{\mu_A(x)}$	
Indeed	$2[\mu_A(x)]^2$ if $0 \le \mu_A \le 0.5$ $1 - 2[1 - \mu_A(x)]^2$ if $0.5 < \mu_A \le 1$	

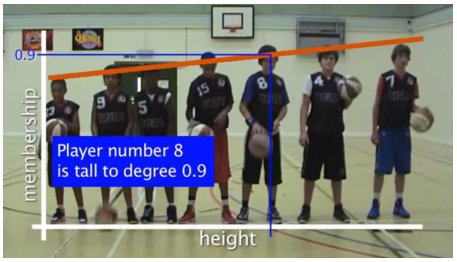
Fuzzy Membership Function



Fuzzy Membership



Fuzzy Membership

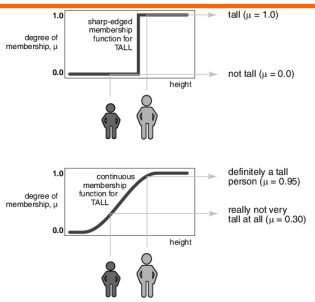


Crisp and Fuzzy membership

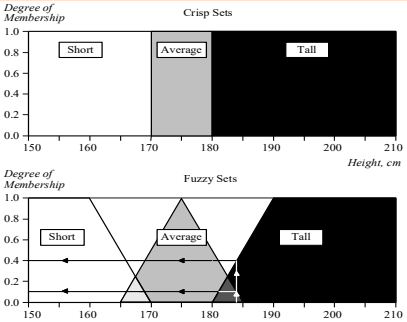
- Consider the figure on the right that compares the crisp and fuzzy membership functions for "Tall"

Membership to set A is often written like this:

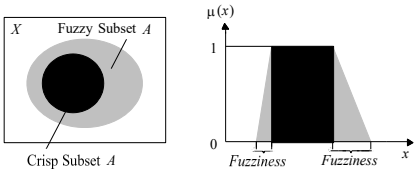
$$A = \{x / \mu_A(x) \mid x \in X\}$$



Crisp and fuzzy sets of short, average and tall men



Representation of crisp and fuzzy subsets



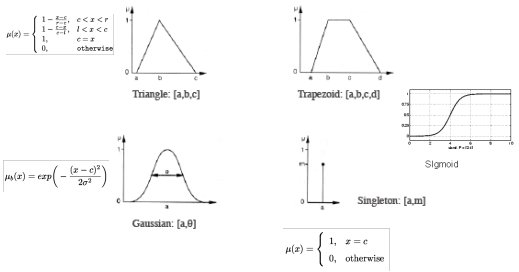
•Typical functions that can be used to represent a fuzzy set are Triangular, Sigmoid, Singleton, Gaussian and Trapazoid.

•In practice most applications use linear fit functions to save in computation time.

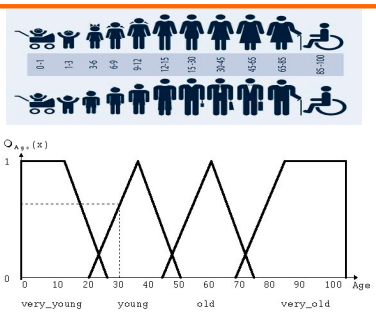
Membership Functions

- A membership function describes the degree of membership of a value in a fuzzy set
- Referred to as MF
  - Also μ (*x*)
    - where *x* is the value being fuzzified
- There are many different types of MF
- Which one to use depends on the problem

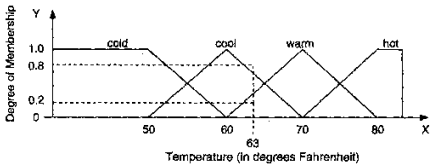
Typical Membership Functions



Age Membership



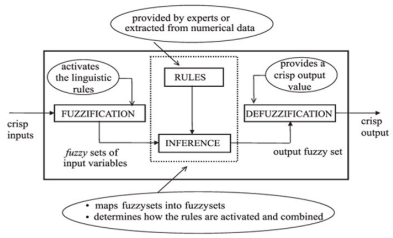
Temperature Membership



Modeling Fuzzy Systems

- Fuzzy system modeling can be pursued using the following steps
  - Step 1: Choose the relative input and output variables
  - Step 2: Determine the number of linguistic terms associated with each input/output variables
  - Step 3: Select a specific type of fuzzy system
  - Step 4: Design a collection of fuzzy if-then rules. To formulate initial rule base, the input space is divided into multi dimensional partitions and then actions are assigned to each of the partitions

Modules of fuzzy logic expert system



Fuzzy Example : Step by Step Approch

- Humans say things like "If it is sunny and warm today, I will drive fast"

- **Linguistic Variables**
- **Membership Functions**
- **Fuzzy Logic**
  - **Fuzzy OR**
  - **Fuzzy AND**
  - **Example**
- **Fuzzy Control**
  - **Variables**
  - **Rules**
  - **Fuzzification**
  - **Defuzzification**

- Linguistic variables:
  - **Cloud Cover:** {overcast, partly cloudy, sunny}
  - **Temp**: {freezing, cool, warm, hot}
  - **Speed**: {slow, fast}

Membership Functions

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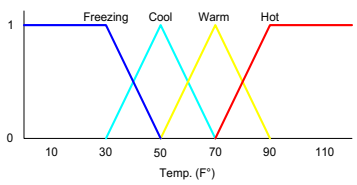
- Temp: {Freezing, Cool, Warm, Hot}
- Degree of Truth or "Membership"



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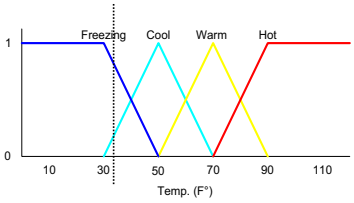
- How cool is 36 F° ?



Membership Functions

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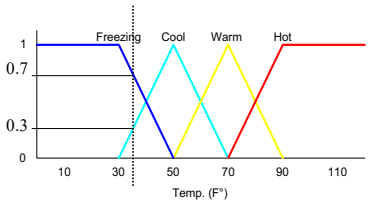
- How cool is 36 F° ?
- It is 30% Cool and 70% Freezing



Fuzzy Logic

- Linguistic Variables
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  - Fuzzy OR
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- How do we use fuzzy membership functions in predicate logic?
- Fuzzy logic Connectives:
  - Fuzzy Disjunction, $\vee$
  - Fuzzy Conjunction, $\wedge$
- Operate on degrees of membership in fuzzy sets

Fuzzy Disjunction

- Linguistic Variables
- Membership Functions
- **Fuzzy Logic**
  - **Fuzzy OR**
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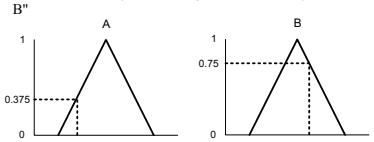
- The fuzzy operation for forming the union of two fuzzy sets A and B on universe X can be given as:
- $A \vee B \triangleq \max(A, B)$

  $\mu_{A \cup B}(x) = max\,[\mu_A(x), \mu_B(x)] = \mu_A(x) \cup \mu_B(x)$, where $x \in X$
- A∨B = C "Quality C is the disjunction of Quality A and B"



- $(A \vee B = C) \Rightarrow (C = 0.75)$

Fuzzy Conjunction

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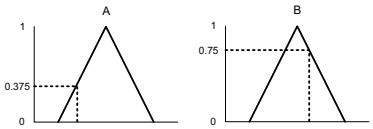
- The fuzzy intersection of two fuzzy sets *A* and *B* on universe of discourse *X*:
- $A \wedge B \triangleq \min(A, B)$

  $\mu_{A \cap B}(x) = min\,[\mu_A(x), \mu_B(x)] = \mu_A(x) \cap \mu_B(x)$, where $x \in X$

  A∧B = C "Quality C is the conjunction of Quality A and B"



$(A \cap B = C) \Rightarrow (C = 0.375)$

Example: Fuzzy Conjunction

Calculate A∧B given that A is .4 and B is 20

- Linguistic Variables
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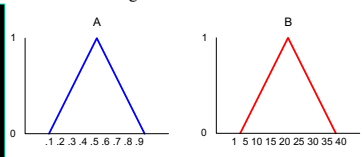

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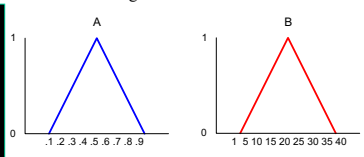

- Determine degrees of membership:

Example: Fuzzy Conjunction

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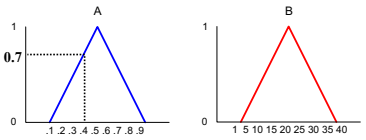
Calculate A∧B given that A is .4 and B is 20



- Determine degrees of membership:
  - A = 0.7

Example: Fuzzy Conjunction

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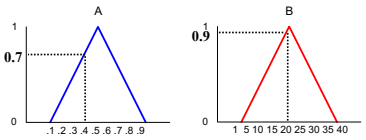
Calculate A∧B given that A is .4 and B is 20



- Determine degrees of membership:
  - A = 0.7 B = 0.9

Example: Fuzzy Conjunction

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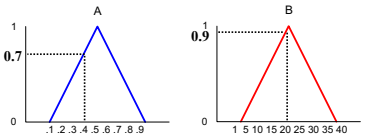

- Determine degrees of membership:
  - A = 0.7 B = 0.9
- Apply Fuzzy AND
  - A∧B = min(A, B) = 0.7

Fuzzy Control

- Linguistic Variables
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  - Fuzzy AND
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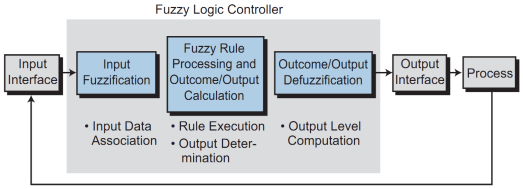
- Fuzzy Control combines the use of fuzzy linguistic variables with fuzzy logic
- Example: Speed Control
- How fast am I going to drive today?
- It depends on the weather.
- Disjunction of Conjunctions

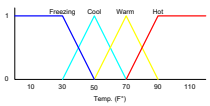
Fuzzy Control



Inputs: Temperature

- Linguistic Variables
- Membership Functions
- Fuzzy Logic
  - Fuzzy OR
  - Fuzzy AND
  - Example
- Fuzzy Control
  - Variables
  - Rules
  - Fuzzification
  - Defuzzification

- Temp: {Freezing, Cool, Warm, Hot}



Inputs: Temperature, Cloud Cover

- Linguistic Variables
- Membership Functions
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  - Fuzzy OR
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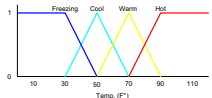
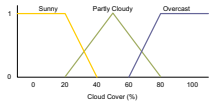


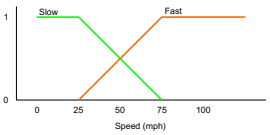
- Cover: {Sunny, Partly, Overcast}



Output: Speed

- Linguistic Variables
- Membership Functions
- Fuzzy Logic
  - Fuzzy OR
  - Fuzzy AND
  - Example
- Fuzzy Control
  - Variables
  - Rules
  - Fuzzification
  - Defuzzification

- Speed: {Slow, Fast}



What is a fuzzy rule?

- Linguistic Variables
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A fuzzy rule can be defined as a conditional statement in the form:

IF x is A
THEN y is B

where x and y are linguistic variables; and A and B are linguistic values determined by fuzzy sets on the universe of discourses X and Y, respectively.

Classical and fuzzy rules?

A classical IF-THEN rule uses binary logic, for example,

Rule: 1
IF speed is > 100
THEN stopping_distance is long

Rule: 2
IF speed is < 40
THEN stopping_distance is short

The variable *speed* can have any numerical value between 0 and 220 km/h, but the linguistic variable *stopping_distance* can take either value *long* or *short*.
In other words, classical rules are expressed in the black-and-white language of Boolean logic.

We can also represent the stopping distance rules in a fuzzy form:

Rule: 1
IF speed is fast
THEN stopping_distance is long

Rule: 2
IF speed is slow
THEN stopping_distance is short

In fuzzy rules, the linguistic variable *speed* also has the range (the universe of discourse) between 0 and 220 km/h, but this range includes fuzzy sets, such as *slow*, *medium* and *fast*.

The universe of discourse of the linguistic variable *stopping_distance* can be between 0 and 300 m and may include such fuzzy sets as *short*, *medium* and *long*.

What is a fuzzy rule?

Fuzzy rules relate fuzzy sets.
In a fuzzy system, all rules fire to some extent, or in other words they fire partially. If the antecedent is true to some degree of membership, then the consequent is also true to that same degree.

Rules

- Linguistic Variables
- Membership Functions
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- If it's Sunny and Warm, drive Fast
  Sunny(Cover)∧Warm(Temp)⇒ Fast(Speed)
- If it's Cloudy and Cool, drive Slow
  Cloudy(Cover)∧Cool(Temp)⇒ Slow(Speed)
- Driving Speed is the combination of output of these rules...

Rules

- Linguistic Variables
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- In a fuzzy system
  - # of variables = V
  - # of membership function (MF) in variables = M_1 , M_2, M_3,....., M_V respectively
  - **No of Rules :**
    - M^V if each variable has M MF each
    - $M_1 \times M_2 \times M_3 \ldots \ldots \times M_V$ else
- **Example**: Let 6 variables with 4 membership functions such as "tiny, small, large, huge".
  - Then the number of possible rules is $4\times4\times4\times4\times4\times4=4^6=4096$
- **Total no of Rules Possible ???**

Example Speed Calculation

- Linguistic Variables
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  - Fuzzy AND
  - Example
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- How fast will I go if it is
  - 65 F°
  - 25 % Cloud Cover ?

Fuzzification: Calculate Input Membership Levels

- Linguistic Variables
- Membership Functions
- Fuzzy Logic
  - Fuzzy OR
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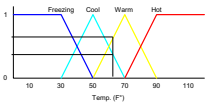
- 65 F° ⇒ Cool = 0.4, Warm= 0.7



Fuzzification: Calculate Input Membership Levels

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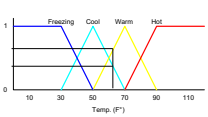
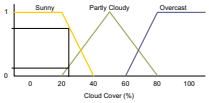


- 25% Cover ⇒Sunny = 0.8, Cloudy = 0.2



...Calculating...

- Linguistic Variables
- Membership Functions
- Fuzzy Logic
  - Fuzzy OR
  - Fuzzy AND
  - Example
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- If it's Sunny and Warm, drive Fast

Sunny(Cover)∧Warm(Temp)⇒Fast(Speed)

$0.8 \wedge 0.7 = 0.7$

⇒ **Fast = 0.7**

- If it's Cloudy and Cool, drive Slow

Cloudy(Cover)∧Cool(Temp)⇒Slow(Speed)

$0.2 \wedge 0.4 = 0.2$

⇒ **Slow = 0.2**

Defuzzification:

- Linguistic Variables
- Membership Functions
- Fuzzy Logic
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  - Fuzzy AND
  - Example
- Fuzzy Control
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  - Defuzzification

- The last step in the fuzzy inference process is defuzzification.
- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.
- The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.

Fuzzy Inference Technique

- Mamdani method
  - In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination.
  - requires us to find the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.
- Sugeno fuzzy inference
  - Michio Sugeno suggested to use a single spike, a singleton, as the membership function of the rule consequent.
  - A singleton, or more precisely a fuzzy singleton, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

Defuzzification:

- Linguistic Variables
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- Converts inferred MF into crisp numbers
- Following defuzzification methods are known to calculate crisp output
  - **Maxima Methods**
    - Height method
    - First of maxima (FoM)
    - Last of maxima (LoM)
    - Mean of maxima(MoM)
  - **Centroid methods**
    - Center of gravity method (CoG)
    - Center of sum method (CoS)
    - Center of area method (CoA)
  - **Weighted average method**

Defuzzification:

- Linguistic Variables
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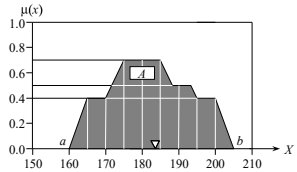
- Converts inferred MF into crisp numbers
- Many different types in existence
- Two common ones
  - Centre of Gravity
    - Where:
      - y is the crisp value
      - K is the number of items in the fuzzy set

$$y = \frac{\sum_i^K \mu(v_i) v_i}{\sum_i^K \mu(v_i)}$$

  - Mean of Maxima
    - Finds the mean of the crisp values that correspond to the maximum fuzzy values
    - If there is one maximum fuzzy value, the corresponding crisp value will be taken from the fuzzy set

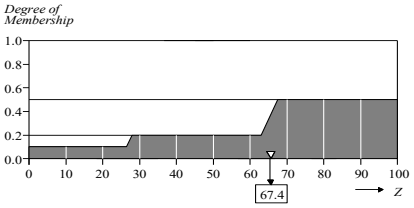
Centroid defuzzification

- Centroid defuzzification method finds a point representing the centre of gravity of the fuzzy set, A, on the interval, ab.
- A reasonable estimate can be obtained by calculating it over a sample of points.



Centre of gravity (COG):

$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$



Defuzzification: Constructing the Output

- Linguistic Variables
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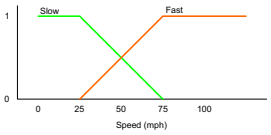
- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

Defuzzification: Constructing the Output

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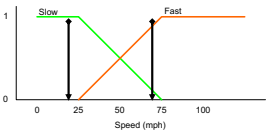

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Defuzzification: Constructing the Output

- Linguistic Variables
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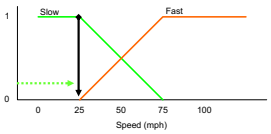

- Speed = weighted mean
  = (2*25+...

Defuzzification: Constructing the Output

- Linguistic Variables
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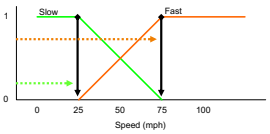

- Speed = weighted mean
  = (2*25+7*75)/(9)
  = 63.8 mph

Notes: Follow-up Points

- Linguistic Variables
- Membership Functions
- Fuzzy Logic
  - Fuzzy OR
  - Fuzzy AND
  - Example
- Fuzzy Control
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  - Rules
  - Fuzzification
  - Defuzzification

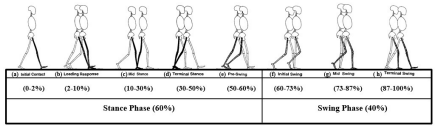
- Fuzzy Logic Control allows for the smooth interpolation between variable centroids with relatively few rules
- This does not work with crisp (traditional Boolean) logic
- Provides a natural way to model some types of human expertise in a computer program

CASE Study :

Automatic Detection Of Abnormal Gait Using Fuzzy System

Abnormality in Gait pattern

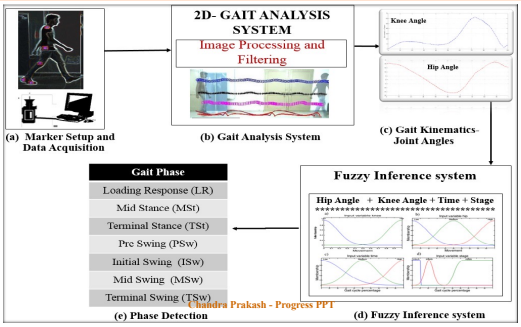
- **Based on Gait kinematics identify phase in gait cycle .**
  - By comparing the normal and current gait cycle phase pattern, healthcare professionals can suggest effective treatment.
  - Recognition of gait cycle phases is extensively useful to spot the time instance at which feedback should be applied for safety and effective response by patient undergoing rehabilitation or physical therapy.



Publication :

- **Chandra Prakash, Kanika, Rajesh Kumar, Namita Mittal, "Fuzzy Logic-Based Gait Phase Detection Using Passive Markers, Chapter in Proceedings of Fifth International Conference on Soft Computing for Problem Solving: SocProS 2015, Volume 1 , pp. 561-572. Springer Singapore, 2016**

Methodology Proposed : Gait Phase Detection



Phase Identification based Fuzzy System

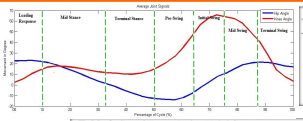


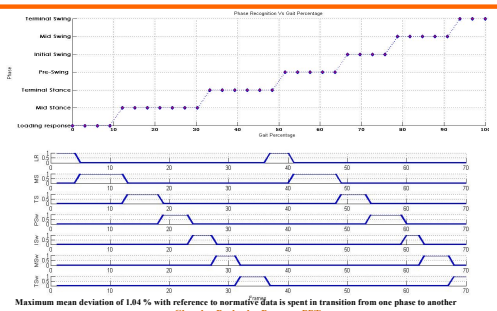
Table 1. Fuzzy Membership Function for inputs parameters

Input	Quantity
Hip	Extension [0 to -30], Low Flexion [0 to 15], High Flexion [15 to 30]
Knee	Low [0-20], Medium [20-60], High [40-70]
Time interval (% of Gait)	Low [0-25], Medium [25-75], High [75-100]
Stage	Loading response (SLR) [0-10%], Mid-Stance (SMS) [10-40%], and Swing Stance (SSS) [40-100%]

Table 2. Set of Fuzzy Rules [1,2,26]

	Hip	Knee	Time	Stage	Gait Phase
1.	High Flexion	Low	Low	SLR	Loading Response (LR)
2.	Not High Flexion	Low	Low	SMS	Mid Stance (MSt)
3.	Low Flexion	Low	Low	SMS	Mid Stance (MSt)
4.	Extension	Medium	Medium	---	Terminal Stance (TSt)
5.	Extension	Not Low	Medium	SSS	Pre-Swing (PSw)
6.	Low Flexion	Not Low	Medium	SSS	Initial Swing (ISw)
7.	High Flexion	High	High	SSS	Mid-Swing (MSw)
8.	High Flexion	Low	High	SSS	Terminal Swing (TSw)

Gait Phase Identification



Maximum mean deviation of 1.04 % with reference to normative data is spent in transition from one phase to another phase.

Advantages of Fuzzy Logic

- Advantages
  - Handles fuzzy terms in language quite easily
  - Logic is simple to implement and compute
  - Very appropriate for device controllers
    - used in anti-lock brakes, auto-focus in cameras, Japan's automated subway, the Space Shuttle, etc

Drawbacks to Fuzzy logic

- How do we define the membership functions?
  - Requires tuning of membership functions
  - there are no learning mechanisms in fuzzy logic
  - what if we have membership functions provided from two different people
    - for instance, what a 6'11" basketball player defines as tall will differ from a 4'10" gymnast
- How do we reconcile the two different fuzzy logics?
- Membership values begin to move away from expectations when chains of logic are lengthy so this approach is not suitable for many KBS problems (e.g., medical diagnosis)
- Fuzzy Logic control may not scale well to large or complex problems
- Deals with imprecision, and vagueness, but not uncertainty

Summary

- Fuzzy Logic provides way to calculate with imprecision and vagueness
- Fuzzy Logic can be used to represent some kinds of human expertise
- Fuzzy Membership Sets
- Fuzzy Linguistic Variables
- Fuzzy AND and OR
- Fuzzy Control

Thank you